Whistling in 1929: Ramsey and Wittgenstein on the Infinite

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Abstract: Cora Diamond has recently criticised as mere legend the interpretation of a quip of Ramsey’s, contained in the epigraph below, which takes him to be objecting to or rejecting Wittgenstein’s Tractarian distinction between saying and showing. Whilst I agree with Diamond’s discussion of the legend, I argue that her interpretation of the quip has little evidential support, and runs foul of a criticism sometimes made against intuitionism. Rather than seeing Ramsey as making a claim about the nature of propositions, as Diamond does, we should understand him as making a claim about the grammar of the logical connectives. Such a view coheres with the extant evidence of the nature of Wittgenstein’s and Ramsey’s 1929 philosophical encounters. It is also compatible with attributing to Ramsey a recognition of Wittgenstein’s distinction and with denying that criticising it is the lesson of the quip.

But what we can’t say we can’t say, and we can’t whistle it either (Ramsey 1929a: 146).

It is widely believed that Ramsey’s quip, the one quoted above, is, in some or other sense, a criticism of Wittgenstein’s Tractarian distinction between saying and showing. Cora Diamond has recently argued that this ‘legend’ is wrong (2011: 340). On her view, the quip expresses a criticism of the fact that the Tractarian account of quantification fails if the world contains infinitely many things. In the *Tractatus*, a universal generalisation is a logical product—a conjunction—of propositions. In the case that the world is infinite, ‘(x) ϕx’ expresses the proposition expressed by ‘(ϕa & ϕb & ϕc & . . .)’, that is, an infinite conjunction of conjuncts, each of which contains the name of a thing. But no-one can state, write down, grasp an infinite conjunction. According to Diamond, it is a thesis of the *Tractatus* that everything that can be expressed—every proposition—can be clearly expressed. So if we cannot clearly say what proposition ‘(x) ϕx’ expresses, then it cannot express a proposition (p. 338). So the Tractarian account is false.

While I believe that Diamond is right both to criticise the legend, and to contend that the target of the quip is the Tractarian view of quantification, I think that the devil is in the detail of her account. In Section 1, I consider the quip’s place in ‘General Propositions and Causality’, which undermines Diamond’s account. In Section 2, I examine her reconstruction of the quip more carefully, and argue that her view is implausible for two reasons: first, it lacks textual support, and second, it lacks (not least of all) charity insofar as it renders
Ramsey’s view an unexplained reaction against the classical conception of the infinite, rather than the consequence of the careful considerations of that distinctive mathematical and philosophical intelligence. In Section 3, I give a better account of the quip, and suggest that by drawing on Wittgenstein’s thought from 1929 we can reconnect the quip to the Tractarian distinction between saying and showing.

Let me begin by getting clear about where I disagree with Diamond’s account, which will also present an opportunity to summarise the Tractarian view. In the *Tractatus*, every proposition is either an elementary proposition (one composed only of names) or a truth function of elementary propositions (a complex proposition). If a generalisation is to count as expressing a proposition, then the proposition that it expresses must fall under one of these categories. At 5.5–5.502 of the *Tractatus*, Wittgenstein introduces the $N$-operator, the role of which is to generate every possible truth-functional proposition from both elementary and complex propositions. The symbol ‘$N(\bar{\xi})$’ is the joint negation of all the propositions indicated by ‘$\bar{\xi}$’, where those propositions can be given in three different ways, either by direct enumeration, by giving a function $fx$, such that the values for ‘$x$’ yield the intended propositions, or by specifying a formal series. What is of greatest importance to note is that how one specifies the propositions which are the value of ‘$\bar{\xi}$’ is not relevant to the meaning of the completed symbol (5.501).

Wittgenstein then introduces the existential quantifier in the case that ‘$\bar{\xi}$’ has as its value all the propositions which are the values of the function $fx$, by writing ‘$N(\bar{\xi}) = (\exists x)fx$’. The universal quantifier ‘$(x)fx$’ is introduced as expressing that proposition which is arrived at by applying the $N$-operator to the function which is produced when a constant is replaced in a proposition to which the $N$-operator has already been applied; this yields a conjunction of propositions in which the denial of each of the conjuncts is denied.

Since, as I mentioned above, it makes no difference to the meaning of the sign how the value of ‘$\bar{\xi}$’ is given, ‘$(x)fx$’ means whatever ‘$N(\sim fx)$’ means, which in turn means whatever ‘$(\sim \sim fa \& \sim \sim fb \& \sim \sim fc \& \ldots)$’ means. The role of the quantifier is, like that of the $N$-operator, simply to form a proposition: a logical product (conjunction) or logical sum (disjunction). And given that the function $fx$ collects together the very same propositions as an enumeration, the proposition formed by binding the variable to a quantifier is the very same proposition as that symbolised by the writing of a conjunction or disjunction. The signs ‘$(x)$’ and ‘$(\exists x)$’ merely indicate what truth function is to be applied to the values of the function containing the variable bound by the quantifier; that is, they indicate the number of times and order in which the $N$-operator is to be applied to those values.

The Tractarian account has much going for it. Besides being an account of quantification which is internally consistent with the Tractarian view that all propositions are elementary propositions or truth-functions of elementary propositions, Wittgenstein’s view provides a natural explanation of the validity of inferences involving quantifier introduction and elimination. If ‘$(x)fx$’ expresses...
a conjunction, then, given the meaning of ‘&’, it is unsurprising that when true, one can infer any proposition of the form ‘fa’. Likewise, given a true conjunction which is exhaustive of the domain and in which each conjunct attributes the same property to each element, the reverse inference is sound. The same reasoning applies to the existential case. Indeed, Ramsey was, until 1929, an adherent of Wittgenstein’s view for precisely these reasons; the Tractarian view was, as far as he could see, the only view capable of explaining the validity of these inference; all other views, including Frege’s functional analysis, leaving that validity mysterious (Ramsey 1927: 49–50).

When a generalization is restricted to the finite, we can understand ‘(x)fx’ as expressing a proposition which might just as well have been expressed by the conjunction ‘(¬fa & ¬fb & ¬fc & . . .)’, treating the ‘. . .’ as an abbreviation for the other finitely-many propositions that are arguments for the truth function. In such cases, it is always in principle possible that such a proposition, no matter how long, might be constructed; the Tractarian account is then just the classical conception of first-order quantification over surveyable domains.

If, however, the variable is not restricted to only finitely many values—if there are infinitely many values that it may take—then it is a different story. In the Tractatus, Wittgenstein does not appear to think that the possibility of an infinite world presents any special problems. But in that case we can no longer interpret ‘. . .’ as an abbreviation in ‘(¬fa & ¬fb & ¬fc & . . .)’. There is no sense to be made of there being some completed proposition that could stand in for ‘(¬fa & ¬fb & ¬fc & . . .)’ that we could write out if only we weren’t so lazy or so finite. On Diamond’s reading of Ramsey, and here is where my disagreement begins, it is to this difficulty that he alludes, namely that Wittgenstein (like Ramsey in earlier papers) simply assumes that his account of quantification carries over to the infinite case. But if there is no such proposition, then there is no proposition that ‘(x)fx’ expresses:

Ramsey’s remark, read in its context, expresses a sharp criticism of his own and Wittgenstein’s accounts of generality: there was a kind of pretence or self-deception internal to those accounts—the pretence that one could somehow specify a conjunction that could not be written out, a conjunction that there was no such thing as writing out. (Diamond 2011: 339)

Diamond is right that Ramsey’s quip is a criticism of his, and Wittgenstein’s, earlier account of quantification, but she locates that criticism in the wrong place, understanding it to be a straightforward rejection of the idea of there being propositions which could, in some reasonably strong sense of ‘could’, never be written down. Instead, seeing Ramsey’s article as evolving out of the way in which he and Wittgenstein had begun to think about the infinite, and especially about the notion of the actual infinite, will make clear what really lies behind Ramsey’s quip.
1. General Propositions and Causality

In ‘General Propositions and Causality’, in which the quip appears, Ramsey draws a contrast between two kinds of generalisations: those that are restricted to a surveyable domain, such as ‘Everyone in Cambridge voted’; and those that are unrestricted and in which the variable may range over infinitely many objects, such as ‘Arsenic is poisonous’ and ‘All men are mortal’. The first kind, which he elsewhere subsumes under the title universals of fact (Ramsey 1928: 140), are adequately treated as conjunctions. In a finite, fixed domain, ‘All \( \psi \) are \( \phi \)' is equivalent to ‘(\( \phi_a \& \phi_b \& \phi_c \ldots \)’ with one conjunct for each \( \psi \).’ As Ramsey points out, what is variable in this kind of generalisation is not the people in Cambridge, since those elements are fixed just in case the domain is fixed, but what is to count as the relevant domain, in this case ‘the limited region of space varying according to the definiteness of the speaker’s idea of “Cambridge”’ (1929a: 145).

The second class, generalisations that Ramsey calls variable hypotheticals, are to be distinguished from the first. The aim of the paper is to give an account of these unrestricted generalisations. But why not think of them, as the Tractatus does, as of the same kind with universals of fact?

Roughly we can say that when we look at them subjectively they differ altogether, but when we look at them objectively, i.e. at the conditions of their truth and falsity, they appear to be the same. (Loc. cit., italics mine)

This distinction between subjective and objective perspectives on unrestricted generalisations is of great importance to my first criticism of Diamond. Subjectively—that is, when we consider the role that generalisations play in our cognition—it is wrong to think of a variable hypothetical ‘(\( x \))f\( x \)' as a universal of fact for three reasons. First (1), we cannot write ‘(\( x \))f\( x \)' as a conjunction—I return to this objection in section 2. Second (2), a variable hypothetical both (2a) does not and (2b) cannot play the role of a conjunction in our thinking.

Taking the descriptive claim (2a) first, Ramsey’s point is that the job of an unrestricted generalisation in thinking is not, as one would expect were it a conjunction, to allow us to make claims about the particular individuals that fall under the class. Consider the claims ‘All arsenic is poisonous’ and ‘Some, but not all, arsenic is yellow’; were these treated as conjunctions, their joint assertion is a Boolean operation on the domain such that the set of yellow arsenic is a subset of the set of poisonous arsenic. But in asserting those claims, we don’t take ourselves to be making claims about any bits of arsenic; while it follows from the claim that if there are any bits of arsenic, some are not yellow, this consequent claim regarding all particulars fails to capture the general content of the original generalisation. Rather, what the generalisation expresses is a lawlike relationship between being arsenic, being yellow and being poisonous. The only cases when we can plausibly be taken to be making the general claim regarding particulars is when the most natural interpretation of the sentence involves a domain restriction, such as ‘All the arsenic on this table is poisonous’ and ‘Some of the
arsenic on this table is yellow’, in which case the generalisations are universals
of fact. This distinction makes itself quite evident in our ordinary talk: ‘All
Queens Regnant are the first-born of monarchs’ is, in the relevant sense, a
perfectly sensible claim, even if there never had been any Queens Regnant, but
an utterance before a table set for tea of ‘All the Queens Regnant on the table
are the first-born of monarchs’ would, rightly, be taken as sign of madness in all
contexts bar that of the logic classroom.

The second part, the modal claim (2b), turns on the thought that ‘a belief of
the primary sort is a map of neighbouring space by which we steer’ (Ramsey
1929a: 146). To take a simple case, my belief that \( p \land q \), along with the belief that
\( q \subset r \), commits me to \( p \), to \( q \) and to \( r \). Those commitments are, of course, already
encoded in the beliefs that I hold, and in that sense they are a map by which I
can steer to or away from other beliefs in the surrounding territory; that I already
hold those beliefs puts me in the position such that, were I presented with
evidence for \( \neg r \), I must either reject it or give up one or both of my current
beliefs, along with any belief in those propositions that I believe because they
follow from them. Surrendering to one of those consequences may in turn make
me loathe to believe \( \neg r \) after all.\(^9\) As those beliefs increase in complexity, so to
does the area of the space for which they are a map. But in the case that the
unrestricted domain is infinitely large, then a generalisation ‘\( (x)fx' \) is an infinitely
long conjunction. Such a belief cannot serve as a map of the surrounding space,
since its consequences are opaque. I could only know the consequences of such
a belief were I able to circumscribe in thought the proposition believed in its
entirety, but, of course, being finite, this I cannot do: ‘our journey is over before
we need its remoter parts’ (Loc. cit.).

The third objection (3) is that, were it the case that my belief that ‘\( (x)fx' \) is an
infinite conjunction, I could never be subjectively certain of it, since, in order to
do so, I would have to assign a belief-degree of 1 to each conjunct, which can
only be done in the finite case. But, as Ramsey notes, it is only in cases where
there is a finite restriction on the domain that we assign a degree of belief to a
generalisation on the basis of its instances; certainty in a variable hypothetical
must therefore derive from some other source.

Ramsey’s subjective objections to subsuming variable hypotheticals to uni-
versals of fact are concerned with our finite capacities, the role of such sentences
in our reasoning, and the ways in which we understand such sentences, and
assess their plausibility. Diamond’s interpretation of the quip places it squarely
within this category, since she is concerned to read it as asserting the non-
existence of propositions that we cannot express. But in the structure of the
article, the quip does not appear here, its natural home were her reading correct;
rather, it occurs directly following Ramsey’s discussion of objective reasons in
favour of the subsumption of variable hypotheticals to universals of fact. This
may suggest that Diamond is on the wrong track.

Objectively (that is, with reference to the conditions under which a generali-
sation is true), ‘\( (x)fx' \) resembles a conjunction in two ways. First, when the
domain is infinite, all finitely long conjunctions of sentences attributing \( \phi \) to the
term of each conjunct will be true just in case ‘(x)ϕx’ is true. And second, it
appears that the condition for the truth of ‘(x)ϕx’ is that each object in the domain
should be ϕ, so that if there are infinitely many objects in the domain, and each
of them is ϕ, then it is the case that ‘ϕa & ϕb & ϕc . . . ’ is true. That is, just as in
the finite case one can instantiate a universal generalisation by means of a finite
conjunction, if we treat a variable hypothetical as being an instance of ordinary
quantification, and thus bivalent, the meaning of the universal quantifier implies
that such an instantiation should be possible in the infinite case as well. But,
writes Ramsey, in that case ‘we are forced to make it a conjunction, and to have
a theory of conjunctions which we cannot express for lack of symbolic power’
(Loc. cit.). It is at this point that the quip appears, immediately following
which Ramsey begins to offer his non-propositional view of variable
hypotheticals, namely that they are rules for judging, and thus not apt for truth
or falsity, but only for agreement or disagreement. I won’t consider the positive
account here.

Note now the following crucial points. First, the only reference to the
impossibility of writing, saying or thinking such a conjunction that Ramsey
makes in his objections to the conjunctive treatment of variable hypotheticals
occurs when considering subjective reasons for disregarding that treatment,
namely in objections 1 and 2b. But even 2b does nothing to impugn the existence
of such conjunctions; a platonist about propositions might hold that the univer-
sal quantifier is the only means by which we can grasp propositions that, though
we cannot express them in full, exist nonetheless and furnish the truth-
conditions for those thoughts. To the platonist, even though we were, for reasons
of finitude, unable to say what proposition ‘(x)fx’ expressed, its guarantor,
waiting in the wings of infinity, would be ever-ready to put meanings in our
mouths. Since an infinite being would share none of our incapacities on this
front, being endowed with both the longevity and the stamina to call that
guarantor by its true name, so we should not allow our merely human limita-
tions to dictate ontology. If this was her view, then the platonist’s task is to
explain how we could ever have grasped the meaning of the quantifier, since
that meaning is necessarily something which outstrips our resources; we are not,
after all, gods. I discuss this further in the next section.

Second, the quip comes after an objection which states that an infinite
conjunction is problematic for logical, or objective, reasons, reasons to do with
the kind of unstateable theory it would force us to adopt for conjunctions.
Ramsey takes it that the theory of conjunctions we would require were the
Tractarian view correct is one that we could not express for lack of symbolic
power; Diamond reads this as a comment on our expressive resources, as
committing us to a ‘theory of conjunctions beyond our expressive powers’ (2011:
338). While the Tractarian theory does commit us to conjunctions that are beyond
our expressive powers (if the domain is infinite), it is unclear at this point why
that should be problematic: arithmetic unproblematically commits us to numer-
als beyond our expressive powers, and the recursive grammar of English to
sentences beyond our expressive powers.
At any rate, Diamond perhaps misreads Ramsey’s admittedly ambiguous sentence: it is not the unstateability of the conjunctions that Ramsey takes to be the unhappy consequence of the Tractarian view (though it is a consequence). Rather, it is the unstateability of the theory of conjunctions that the view commits us to.

2. Impossibility and the Infinite

Wittgenstein later made several remarks about his Tractarian treatment of quantification, and the difficulties presented for that treatment by an infinite domain, remarks which were recorded by Moore. There are, in particular, two which are worthy of attention here. Moore writes (A) that Wittgenstein said he had made the mistake of supposing that an infinite series was a logical product—that it could be enumerated, though we were unable to enumerate it. (Moore 1955: 4)

And (B):

There is a most important mistake in [the] Tractatus . . . I pretended that [a] proposition was a logical product; but it isn’t, because ‘. . .’ don’t give you a logical product. It is [the] fallacy of thinking $1 + 1 + 1 \ldots$ is a sum. It is muddling up a sum with the limit of a sum (Proops 2013). 10

It is quite natural to read these gobbets as commenting on what propositions there are or could be, as seeing a problem with the thought that there might be such a thing as a proposition which could never be grasped or expressed by a finite being, and thus as being as one with Diamond’s view of the quip. Assuming that the Tractatus account of propositions cannot allow for the existence of propositions which go beyond our capacities to express them, even in principle, such an interpretation requires that we either abandon the idea that an instance of quantification expresses a proposition, or we find an alternative theory of the quantifiers which still allows that they express propositions without being equivalent to truth-functions of propositions. According to Diamond (p. 338), Ramsey makes implicit appeal to a line from the Preface of the Tractatus in order to discharge that assumption: ‘What can be said at all can be said clearly, and what we cannot talk about we must pass over in silence’. By the Tractatus’ own lights, if we cannot say, and say clearly, what proposition it is that an instance of quantification over an infinite domain expresses ‘without fudging’, then we must accept that there is no such proposition. Having thus established that the unrestricted universal quantifier cannot be interpreted as a logical product if the domain is infinite, Ramsey’s strategy is to endorse the former of the two available options: generalisations over an infinite domain do not express propositions at all.

Diamond’s account relies upon a single line in the Preface of the Tractatus. While this would be sufficient to warrant the claim that the evidence upon which
she relies is slim, the situation is much worse, for her reliance on a particular interpretation of that line renders her argument a *petitio principii* within her broader dialectical position.

In order to explain why this is so, I need to put the debate about the quip into a wider context. Why, someone might ask, do we care what Ramsey thought? Whether or not we have got Ramsey right is perhaps a matter of merely local interest. But the stakes are raised when the quip is put, as it frequently is, to exegetical use in order to justify an interpretation of the *Tractatus* which imputes to Wittgenstein a particular conception of unsayability, namely one which committed him to the existence of ineffable thoughts, contents or insights which can be grasped but which cannot be said.\textsuperscript{11} The criticisms contained in the quip as interpreted according to the legend are only effective if there is good reason to think that what Ramsey took Wittgenstein to be doing is what Wittgenstein was indeed doing, and at this point the indisputable fact of Ramsey’s close relationship with Wittgenstein is wheeled out to do some argumentative work. Everyone accepts that Ramsey holds a special position when it comes to understanding Wittgenstein:

What [Ramsey] says about Wittgenstein is always worth taking seriously, far more than anything said by anyone else who was in contact with Wittgenstein in the years before 1930. (Diamond 2011: 336)

Diamond is, famously, someone who denies that the *Tractatus* is committed to ineffable thoughts or insights, and that failing to take the nonsense of the *Tractatus* to be ‘real nonsense, plain nonsense’ is ‘chickening out’ (Diamond 1988: 7). Since Diamond wants to put her interpretation of Ramsey’s quip to hermeneutical use, too, if not directly to support her views on Tractarian nonsense then at least to remove from her opponents an important piece of evidence for the alternative, she does not dispute this conception of Ramsey’s role in Wittgenstein scholarship. We can then construct the following argument:

(1) If Ramsey believed that, in the *Tractatus*, Wittgenstein was doing such-and-such, then (given Ramsey’s unique position among early readers of Wittgenstein), that fact should be regarded as very good evidence that he was doing such-and-such in the book.

(2) Ramsey’s quip expresses a general criticism of the *Tractatus*, namely that Wittgenstein attempted to evade his own conclusion that his insights were unsayable, by trying to get them across in some other way.

(3) Ramsey’s quip provides very good evidence that Wittgenstein did attempt to evade his own conclusion that his insights were unsayable, by trying to get them across in some other way (Diamond 2011: 336).

Wishing to resist (3), Diamond must undermine (2). But in order to do so, she makes appeal to the very conception that she wishes to defend. The line in question is, of course, open to Diamond’s reading, but it can equally be seen as endorsing the robust conception of the unsayable that Diamond wishes to resist.
attributing to Wittgenstein. An adherent of the traditional view would say that Wittgenstein is merely mirroring in the Preface what is expressed at the end of *Tractatus*, and would interpret him as saying that there is nothing to be gained but confusion from any attempt to say what can only be shown—that is, to say what is unsayable. But since Diamond makes use of Ramsey’s argument in order to argue against the view that Wittgenstein endorsed the traditional distinction, she cannot legitimately put that conclusion to work in interpreting Ramsey’s argument. What Diamond needs are independent reasons for attributing her version of the quip to Ramsey, reasons that she does not provide.

Further, the view that Diamond attributes to Ramsey faces some familiar difficulties. In particular, it is unclear what is to count as ‘sayable’ in Diamond’s sense, i.e. what it is to say of something that it *can* be written down. The *Tractatus* is committed to the result that there are $2^\kappa$ complex propositions, where $\kappa$ is the (possibly infinite) number of elementary propositions. But in that case worries about expressibility enter the scene long before we consider the infinite; the number of observable atoms in the universe is estimated to be no greater than $10^{87}$, and if each were the most basic syntactic unit, there would still not be enough to write down even one of many finite conjunctions of merely moderate length. Taking Ramsey’s example, would the contingency of Cambridge’s enjoying a population of $10^{87}$, rather than $10^5$ force upon us a different account of the meaning of ‘Everyone in Cambridge voted’ simply because the world might be over before everyone’s name had been uttered?

What would be required to resolve this difficulty would be a principled distinction which made clear why the manner in which it is impossible to write down (survey, express or understand) a finitely long conjunction of unutterable length differed from the manner in which it is impossible to write down (survey, express or understand) a conjunction of infinite length. That is, why can there be no graspable conjunctions, no propositions, of one kind—the infinitely long—while there are graspable conjunctions of the other kind—the hugely-but-merely finitely long? If no such distinction is made out, then we risk characterising Ramsey as akin to a strict finitist, that is, as believing that, for instance, what propositions there are (or what numbers there are) is to be decided by the actual limitations on human cognition or activity, as opposed to limitations that stem from what is in principle beyond our ability to write down, survey, express or understand. I can find no reason to suppose that he held a strict finitist view at any time. For instance, in ‘General Propositions and Causality’, one way in which Ramsey brings out, in the course of objection 2b, the difference between the two cases of generalisation is by appeal to a belief as a map which ‘remains such a map however much we complicate it or fill in details. But if we professedly extend it to infinity, it is no longer a map; we cannot take it in or steer by it’ (p. 146). So no matter how we complicate a finite proposition—how far we extend it, how many conjuncts we add to it, even, presumably, if we extend it beyond the stretch of any idealised mortal’s lifespan—it remains a map with a role to play in inference and cognition. When we entertain the thought that there are infinite propositions, however,
we must admit that we have made a mistake; such a proposition simply could not play the role that is required of it.

The kind of distinction that we require in order to make sense of Diamond’s use of ‘sayable’ is one which would clearly distinguish her meaning from what Russell called the merely medical impossibility of completing an infinite operation:

Miss Ambrose says it is logically impossible to run through the whole expansion of $\pi$. I should have said it was medically impossible. She thinks it logically impossible to know that there are not three consecutive 7s in $\pi$. But is it logically impossible that there should be an omniscient Deity? (Russell 1936: 143)

The object of dispute between Ambrose and Russell is how to circumscribe our, perhaps idealised, capacities in a way that is philosophically relevant in an account of mathematical truth. Ambrose sees a deep incoherence in the thought of the completion, by even an ideal being, of the decimal expansion of $\pi$. Russell sees only a contingent limit, the flimsiness of which is revealed by the possibility of God’s timeless eye. The very same dispute arises in the debate between platonists and those intuitionistic philosophers of mathematics, such as Dummett, who deny the coherence of the actual infinite, that is, the existence of a mathematical object which is the completion of an infinite process. And just as Diamond’s Ramsey is threatened with the prospect of strict finitism unless that circumscription can be adequately made out, so too is it sometimes thought that the intuitionist faces the same challenge.

There is, however, an important difference between the two cases. Whereas Dummett’s intuitionism arises from a general background philosophical theory about meaning, a theory which furnishes him with the capacities to fend off the platonist challenge at least to his own satisfaction, Diamond provides us with no similar means of sketching a defence of Ramsey’s view, even in principle. Instead, we have only the sense of a kind of mental balking at the idea of an infinitely-long conjunction (but not, presumably, at very large finite conjunctions), without any attendant theory by which to adjudicate whether such resistance is warranted.

Consider the intuitionist response to the Russellian challenge. For a platonist, grasp of the meaning of a mathematical statement is a matter of grasping its truth conditions. The intuitionist, denies that we can have an understanding of the conditions under which some statement, making putative reference to a completed infinity, could be true; and if we, per impossibile, had such an understanding, we would yet be short of the means by which we could display it. Unlike the strict finitist, who denies the coherence of a statement asserting a predicate of some natural number $n$ which is not such as to be established (or refuted) by a proof that we could actually construct, the intuitionist holds that we can grasp the meaning of such a statement only if we have a grasp of what would constitute a proof of that statement, where a proof is a completable operation, even if such a proof would outrun our practical capacities. Now here
we have precisely the same difficulty as confronts Diamond’s view, namely the specification of what is to count as completable.

If the intuitionist is correct, and our grasp of the meaning of a mathematical statement is to be given in terms of completable operations upon the natural numbers, then she is going to be able to make no sense of anyone’s grasping the truth of a mathematical statement which makes reference to the completion of a process which is, by definition, incompletable. In general (but not always), statements which quantify over all of the natural numbers, for instance, will be ruled out since their meaning would have to be given in terms of truth conditions apprehension of which is not possible given our capacities, or some admissible extension of them, where it is yet to be decided what is to count as ‘admissible’.

The difficulty is paralleled by the case of infinitely long conjunctions of propositions. If ‘(x)fx’ expresses such a conjunction, then we are owed an explanation of how it is that we grasp the conditions of its truth, given that its truth-value is a function from the truth-values of infinitely many conjuncts. For any finitely long conjunction, we have, of course, the requisite understanding—we have grasped its truth conditions if we grasp the truth conditions of each of the conjuncts. But in the case of an infinitely long conjunction, that process—of grasping the truth conditions of each conjunct—is incompletable, and so, the argument goes, we have no conception of its truth conditions, and thus no grasp of its meaning.

Diamond provides us with no reason to think that Ramsey held any such detailed theory, and so the weight of Diamond’s interpretation falls upon the circumscription of the relevant notion of sayability, where the desideratum is that it should not collapse his view into an absurdly strict finitism. Indeed, according to Diamond, the only theory that Ramsey held is the one that she attributes to the Wittgenstein of the Preface of the *Tractatus*, and yet she fails to provide any evidence to support this claim. Absent some deeper account, however, we are left floundering, unable to assert that the restriction imputed to Ramsey amounts to anything more than an uncharacteristic attack of the nerves with regard to the infinite. Let us seek, then, for an alternative understanding.

3. Saying and Showing

My task will be to make sense of Ramsey’s remark that the Tractarian account of quantification saddles us with ‘a theory of conjunctions that we cannot express for lack of symbolic power’, and not, as Diamond holds, merely conjunctions that we cannot express for lack of expressive power. In the *Philosophical Remarks*, Wittgenstein writes that ‘you can’t talk about all numbers, because there’s no such thing as all numbers’ (Wittgenstein 1975: §124), and that ‘it isn’t just impossible “for us men” to run through all the natural numbers one by one; it’s impossible, it means nothing’ (§124). The latter claim stands against strict finitism, but still leaves us short of a theory. Why should we say that there is no such thing as all the numbers?
Wittgenstein’s concern in Section XII is to investigate the ways in which the concept of infinity is tied to a conception of possibility which is neither logical nor physical, but rather grammatical. Talk of infinity in the case of the natural numbers is to be understood as talk about the nature of the rules of the symbolism for forming a particular kind of expression, the numerals. Asserting of the numbers that they are of infinite cardinality is an attempt to say of the rules for forming expressions of number that they block a certain possibility with regard to the construction of numerals, namely that there should ever be an end to such construction. And it should be noted that the impossibility of completion is not to be understood as relating to the finite capacities of human beings, nor the finite magnitude of the universe’s material, spatial or temporal resources:

The rules for a number-system—say, the decimal system—contain everything that is infinite about the numbers. That, e.g., these rules set no limits on the left or the right hand to the numerals; this is what contains the expression of infinity. Someone might say: True, but the numerals are still limited by their use and by writing materials and other factors. That is so, but that isn’t expressed in the rules for their use, and it is only in these that their real essence is expressed. (§141)

The kind of possibility with which infinity is associated—a grammatical possibility—is a possibility contained within the structure of our symbolism. As such, it is a mistake to think of the grammatical possibility of the unconstrained nature of numerical expressions as being akin to the kind of possibility which is to be contrasted with the actual. Wittgenstein noted that the word ‘possibility’ was not perhaps the most perspicuous in this context since ‘someone will say, let what is possible be actual’ (§141), and it is precisely this move which leads to error. If we consider the function \( m = 2n \), Wittgenstein’s account requires him to say that it follows from the rules for the symbolism that, for any numeral \( n \), \( m = 2n \) correlates it with a number \( m \); that is, that \( 'm = 2n' \) ‘contains the possibility of correlating any number with another’. But that is not to say that there is any sense to be made of the claim that \( m = 2n \) does correlate every number with another: ‘In the superstition that \( m = 2n \) correlates a class with its subclass, we merely have yet another case of ambiguous grammar.’ The slide from one sense in which \( m = 2n \) correlates each number with another, namely that the rules of the symbolism permit the construction of a new symbol as value from any other as argument, to the other, in which \( m = 2n \) carves up the set of numbers into ordered pairs, results from the way that Wittgenstein’s notion of a grammatical possibility conflicts with the ordinary conception of possibility, which can be contrasted with the actual. Ordinarily, if it makes sense for me to say that it is possible that such-and-such could be the case, it also makes sense for me to say, perhaps falsely, that such-and-such is the case (§142). But in the case of a grammatical possibility, while it is possible that there are infinitely many numbers, insofar as there is no grammatical constraint on the number of numerals allowed by the symbolism, it does not, merely on the basis of the sense of this claim, make sense to say that there are infinitely many numbers. The
inference from the former to the latter is illegitimate, a case of our being led astray by the language that we use to describe the rules that govern our talk of number.

The notion of an actual infinity is then a characteristic example of a grammatical error, namely that of conflating a grammatical possibility with the more robust modal notion. Consider the natural number series, which Wittgenstein takes to be given by the symbol ‘(1, \(x, x + 1\))’. This symbol encodes the rule that governs our talk of number, and the occurrence of the variable ‘\(x\)’ expresses that feature of the concept number, namely that no matter its value, no matter how large the number, we can always form a further symbol ‘\(x + 1\)’ which yields a greater number. The symbol contains the possibility of infinitely many numbers since the rule that it expresses places no bound on the construction of numerical symbols. We can, of course, legitimately characterise the natural numbers as being infinite, so long as we are clear that we are expressing a feature of the rules governing our symbolism. The slide to be avoided is from using the term ‘infinite’ in connection with some concept in order to ‘exclude nothing finite’ to using the term in order to characterise the cardinality of the extension of that concept; that is, the use of that term as standing for a number (§138).

Evidence that Ramsey and Wittgenstein at least discussed this view of the infinite is provided by a document, written in German, contained amongst Ramsey’s papers. It has been suggested that Wittgenstein dictated these notes to Ramsey in preparation for a talk on infinity that he gave to the Aristotelian Society in 1929 in place of ‘Some Remarks on Logical Form’, which he refused to deliver (McGuinness 2006: 24). The document consists of notes in German, interspersed with occasional asides in English, and it concludes:

Infinite possibility is represented by a variable which is such that the possibility of its being filled is without limit; and the infinite may not occur in the proposition in any other way. [Die unendliche Möglichkeit ist durch eine Variable vertreten die eine unbegrenzte Möglichkeit der Besetzung hat; und auf andere Art darf das Unendliche nicht im Satz vorkommen.]

For Wittgenstein in 1929, mathematical objects are nothing more than the permanent possibilities of certain symbolic operations: our grasp of the meanings of mathematical statements can only be explicated in terms of the possibilities of such operations, governed by the appropriate grammatical rules, such that our grasp of a mathematical concept is wholly exhausted by our grasp of a certain rule.

Returning to the two quotes from Moore, it should now, in the light of what has been said about the Remarks, be clear that Wittgenstein’s criticism of the Tractarian account of quantification is not located where Diamond suspects. (A) is not an objection to the idea of the possibility of an infinite enumeration which we could never succeed in completing, but rather that such a conception, that of a completed infinite, is the result of a grammatical mistake. This objection is even clearer in (B): the fallacy of thinking that ‘1 + 1 + 1 . . .’ is a sum is the slide from
noting the possibility of infinite extensibility contained in the symbol to thinking that there is a completion of the symbol, some number that its completion would represent.

If we think of a universal quantification over an infinite domain as expressing an infinitely long truth-function, namely conjunction, then we must think of our grasp of the proposition that it yields as being the completion of an infinite process. But the grammar of conjunction, even as it is discussed in the *Tractatus*, rules out such a possibility. There, truth-functions are called operations, and what differentiates an operation from a function is that, while an operation can take as its argument whatever its value is for some other arguments, a function cannot. Operations, unlike functions, are such that there is no grammatical restriction upon their completion; it is presumably for this reason that Wittgenstein saw them as a fitting base for our grasp of number.

To say that there are infinitely many arguments for some truth-function such as conjunction is merely to acknowledge that the grammar of the symbolism for that operation fails to constrain it by imposing a completion. It is always possible that a conjunction should take a further proposition as a conjunct, without plunging from sense into nonsense. But it is a mistake to think that, since it is possible that the operation of conjunction may be infinitely iterated, there is any sense to the thought of that possibility’s being actualised, of the notion of an infinitely long conjunction having any sense at all. That is why it is a mistake to think of such a truth-function yielding a truth-value, just as it is a mistake to infer from the grammatical possibility of ‘1 + 1 + 1 . . .’ being infinitely extensible to its being a sum. Universal generalisations over infinite domains cannot thus be truth-functions of atomic propositions, and thus cannot be propositions at all.

In the *Tractatus*, Wittgenstein envisages the application of the $N$-operator to a class of propositions as a single operation; in the case where the domain is infinite, this would mean the single application of an operation to an infinite class of operands. It might be thought that this way of conceiving of the $N$-operator avoids the difficulties to do with the grammar of conjunction that I have raised above. After all, on this conception, no infinite process is invoked. This point, however, evades the difficulty raised by using the expression ‘single application of an operation to an infinite class of operands’ without saying to what this amounts. Consider the natural numbers: I might say that applying the function ‘+’ to a set $N$ consisting of every number was not to invoke an infinite process as it is the single application of an operation, albeit to an infinite class. But what would be required of me to grasp that sum, were it even recognisable as a sum? Likewise, what would be required of me to grasp the proposition expressed by the single application of the $N$-operator to an infinite class of operands? Surely nothing less than the completion of an infinite process, namely the grasp of the truth-conditions of infinitely many finite conjunctions?

The difficulty does not lie with the specification of an infinite class of operands: in that case, while I cannot enumerate the operands, I may specify them by giving a function, $f$. The difficulty lies, rather, with the attempt to perform an operation upon the members of that class, and the attendant
expectation that what results will be something complete, a proposition. There is nothing wrong with saying that it is possible to conjoin infinitely many propositions, so long as what is meant is that the grammar of conjunction imposes no completion upon its use, just as there is nothing wrong with saying that it is possible to sum infinitely many numbers, so long as what is meant is that the grammar of ‘+’ imposes no completion upon its use. What is wrong, however, is to think that either possibility is of the kind which is to be contrasted with the actual. In the latter case, this leads to the conclusion that the equation has a solution of a certain sort, and in the former to the claim that the meaning of a universal generalisation might be an infinite conjunction. The confusion rests upon an illegitimate slide between two meanings of the word ‘possible’, between the grammatical use and the modal use, the first of which is independent of the notion of the actual, and the second of which is not.

What, then, are we to make of Ramsey’s quip? Recall that his complaint, prior to the quip, was that the Tractarian account of quantification lumbers us with a theory of conjunction ‘which we cannot express for lack of symbolic power’ (1929a: 146). Assuming that he had been at all swayed by the thoughts Wittgenstein later expressed in the Remarks, Ramsey’s comment can be understood as suggesting that, were the Tractatus account of quantification correct, in the case that the domain is infinite it would commit us to the thought that our understanding of the conjunction outruns our grasp of the rules governing the symbolism, that there is somehow more to its meaning than what is given by a grasp of those rules. But if that were so, we would be unable to state our understanding of the conjunction by appeal to cases of its use since, ex hypothesi, its meaning extends beyond any of those possible occurrences. We would then be required to give a theory of quite a different character, perhaps platonic, in which our grasp of the truth-function as expressed by our symbolism is only a partial grasp of the referent of that symbol, the rest of our understanding being filled in by some other route altogether. But, of course, that is to leave far behind what both Wittgenstein and Ramsey considered to be an important insight of the Tractatus, namely that the logical constants are not referring expressions (4.0312).

One Tractarian method of arguing that our grasp of conjunction goes beyond the rules governing its use would be an appeal to its being shown that that is the case. Recall the argument that Ramsey gave in ‘Facts and Propositions’: there, he argues that the Tractarian account is the only one which can explain the fact that from ‘(x)fx’ we can always infer ‘fa’. That is, our inferential practice shows that a certain view of quantification is the correct one. When the domain is infinite, we can, assuming that ‘(x)fx’ has a sense, always infer ‘fa’, so why not argue that, while we cannot state the theory of conjunctions that underwrites the inference, its being so underwritten shows that ‘(x)fx’ is a conjunction? And, recall, it is indeed this feature this Ramsey takes to be the thought which, when variable hypotheticals are considered objectively, beguiles us into asserting a theory of conjunctions beyond our symbolic power.

One response might be that this is just not the right kind of ‘showing’ to do the work required of it. The traditional understanding of that distinction holds
that when one tries to say what has been shown, one produces nonsense. But in this case we cannot even produce the kind of nonsense we might mistake for sense. More than that, this line of argument would be entirely unsatisfactory by the *Tractatus*’s own lights, for its conclusion would imply the platonism described above, a view so much at odds with its own theory. That view is retrograde, and Ramsey’s quip serves to warn us against the kind of argument that might otherwise be so tempting, and indeed tempted him on earlier occasions, for its conclusion drives us back to a troubled theory of logic, and of meaning, from which the *Tractatus* aimed to help us to escape.

If I am right, can we draw any morals regarding the distinction between saying and showing? It would appear that Ramsey did recognise the distinction, but nothing in my reconstruction of his objection entails comment on ineffability. Rather, what he is criticising, very much in the realistic spirit (1929a: 160), is a theory which attributes to us the grasp of a meaning, whether for logic or for number, that we can never, even in principle, exhibit. Faced with the question ‘how can I know what “\((x)fx\)” means?’, the purveyor of the target theory can only wave his hands and whistle.18

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NOTES

1 The same is widely believed of another of Ramsey’s remarks: ‘Or else it is a disposition we have to check, and an inquiry to see that this is so; i.e. the chief proposition of philosophy is that philosophy is nonsense. And again we must then take seriously that it is nonsense, and not pretend, as Wittgenstein does, that it is important nonsense!’ (Ramsey 1929b: 1).

2 See Geach 1981 and 1982 for a more detailed exegesis of the $N$-operator, and a proposal for a more perspicuous representation of it in cases of multiple generality.

3 Morris (2008: 217–21) thinks that infinite domains raise no problems for Wittgenstein’s account of quantification. That is because while he correctly observes that Wittgenstein does not define the quantifiers in terms of conjunction or disjunction, he goes too far in ignoring the crucial claim that there is no difference between a proposition formed by applying the $N$-operator to a function and applying it to an enumeration of propositions. Morris also claims that Ramsey agreed with his assessment that the account was unproblematic for the infinite domain, citing ‘Foundations of Mathematics’ (1925) and ‘Mathematical Logic’ (Ramsey 1926). But on Morris’s view, since a universal generalisation does not, strictly speaking, mention any particular things, it is wholly unspecific. That this is not the view with which Ramsey agreed is clear from what Ramsey says in ‘Facts and Propositions’: ‘Besides, that $a$ is involved in the meaning of “For all $x$, $fx$” can be seen from the fact that if I say “For all $x$, $fx$”, and someone replies “not-$fa$”, then, even though I had not before heard of $a$, he would undoubtedly be contradicting me’ (1927: 50).
Morris’s view is one which, it seems to me, violates the whole notion of a proposition in the *Tractatus*, namely that every proposition be an atomic proposition or a truth-function of atomic propositions, since his view fails to say what proposition a generalisation is.

4 For instance, he considers the possibility that the world might consist of an infinite number of objects, but holds that this does not affect his arguments for logical atomism: ‘Even if the world is infinitely complex, so that every fact consists of infinitely many atomic facts, and every atomic fact is composed of infinitely many objects, there would still have to be objects and atomic facts’ (4.2211). And in his short discussion of Russell’s Axiom of Infinity (5.535), he regards the issue of whether the world is infinite or not as one that would be decided by whether or not language contained an infinite supply of names. That is, what is the case with regard to the finitude or infinitude of the world he regarded as something which would be shown by language, namely by the range of elementary propositions that are expressible in it. Since this is something shown, it cannot be said, and since the Axiom of Infinity attempts, either truly or falsely, to say something which is unsayable, it must be nonsense; that it is nonsense is evident when one considers that any statement of the Axiom of Infinity must involve the concept word ‘object’.

5 Only a few years earlier, Ramsey thought not only that the *Tractatus* was committed to such propositions, but that allowing them was an ‘important innovation’: ‘Mr Wittgenstein has perceived that, if we accept this account of truth functions as expressing agreement and disagreement with truth-possibilities, there is no reason why the arguments to a truth-function should not be infinite in number. […] Of course if the arguments are infinite in number they cannot all be enumerated and written down separately; but there is no need for us to enumerate them if we can determine them in any other way, as we can by using propositional functions’ (Ramsey 1925: 170–71).

6 There is another famous criticism, addressed by Ramsey, of Wittgenstein’s account, namely that ‘\( (f_a & f_b & \ldots & f_z) \)’ cannot be equivalent to ‘\( (x) f_x \)’ since it would have to include as an addendum ‘and \( a, b, \ldots, z \) are everything’ (Ramsey 1927: 50). Ramsey there thinks that this objection is answerable, but as neither Diamond nor I find it relevant to the present discussion, I do not pursue it here.

7 It might be objected that ‘\( \forall \psi \phi \)’ can be interpreted over an unrestricted domain, in which case, if that domain is infinite, it cannot be represented as a finite conjunction. But I take it that Ramsey is thinking of the most natural reading of such generalisations as implying a restricted reading.

8 In ‘Philosophy’, Ramsey discusses the importance of variable hypotheticals in showing that ‘in this part of logic we cannot neglect the epistemic or subjective side’ (1929b: 6).

9 The picture is more complex if I hold these beliefs in degrees; in that case, the map that one set of beliefs provides will be one that assigns a continuum of related values to a possible configuration of belief states.

10 Proops quotes this passage from Moore’s unpublished notes: *Moore Archive*, 8875, 10/7/7, 37, entry for November 25. Compare Ramsey’s view on the matter in 1925, where he is objecting to precisely this comparison made as a criticism against the Tractarian view of quantification by Hilbert: ‘Thus the logical sum of a set of propositions is the proposition that one at least of the set is true, and it is immaterial whether the set is finite or infinite. On the other hand, an infinite algebraic sum is not really a sum at all, but a limit, and so cannot be treated as a sum except subject to certain restrictions’ (p. 170n).

11 Two exemplars are Hacker and Mellor. Hacker both interprets each of the quotes as critical of Wittgenstein’s distinction, and uses the fact that it was Ramsey who wrote these
things as evidence for interpreting in the *Tractatus* a commitment to ineffable insights (Hacker 2000: 355). Mellor reads the quip as not only a criticism, but a ‘deep objection’ to the *Tractatus* (Ramsey 1990: xvi). He further implies that, since Ramsey ‘was instrumental in persuading Wittgenstein to abandon’ the whole approach of the *Tractatus*, Wittgenstein must have been moved by the objection, and so must have held the view being objected to. Similar approaches are taken to other remarks that crop up in Ramsey’s papers. For instance, in ‘Philosophy’, he writes that philosophical neglect of the concept of meaning risks putting us ‘in the absurd position of the child in the following dialogue: “Say breakfast.” “Can’t.” “What can’t you say?” “Breakfast.” ’ (1929b: 6). Gordon Baker interprets this as the claim that Wittgenstein’s distinction between saying and showing results in absurdity, but argues that this results from a misunderstanding of Wittgenstein’s reasoning on Ramsey’s part (Baker 1988: 84n).


13 Archive number 004-23-01 in the Ramsey Collection at the Hillman Library, University of Pittsburgh.

14 I think it likely that this line of thought marks an important stepping stone to Wittgenstein’s later rule-following considerations. What we have here is a disavowal of a platonistic conception of number and of logic in favour of a grasp of something finite, namely a rule for a certain symbol. In his conversations with Waismann and Schlick in January of 1929, it is interesting to note that Schlick especially was keen to pursue the question of the validity of such a rule: ‘How do I know that precisely these rules are valid and no others? Can I not be wrong?’ (Waismann et al. 1979: 73–81). Postulating that mathematical truth is relative to a particular rule leaves us with something halfway between platonism and the ideas of the *Investigations*, namely a view which continues to render our practices answerable to some external, sempiternal standard, our grasp of which remains mysterious.

15 This is, in essence, Wittgenstein’s response to Russell’s paradox. Note, too, that on this account, Russell’s paradox is not a logical error, but a grammatical one.

16 In the *Tractatus*, the natural numbers are indices on the application of an operation to an argument; the infinitude of the natural numbers thus depends on the possibility of an operation’s forever taking its value at stage \( n \) as its own argument at stage \( n + 1 \).

17 My thanks to an anonymous referee for making this point.

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REFERENCES

Whistling in 1929


